Unranked Tree Automata with Sibling Equalities and Disequalities

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Lehrstuhl für Informatik 7
RWTH Aachen

ICALP 2007
Wrocław, 9–13 July 2007

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Motivations(1): Tree Automata with Subtree Equalities

Finite (bottom-up) tree automata:

- ranked alphabet: each symbol has fixed rank/arity
- nice closure properties, determinizable
- decidable emptiness

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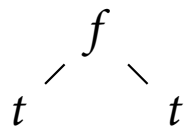
But language of trees  is not recognizable

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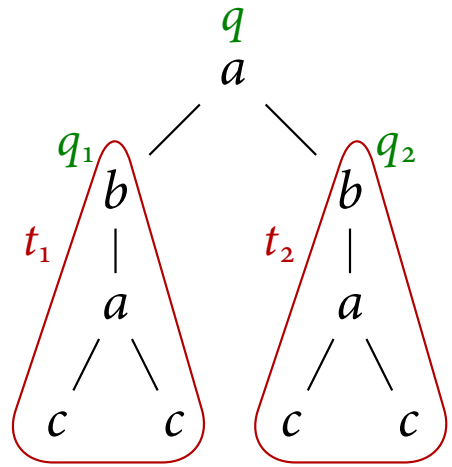

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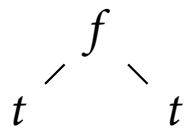
But language of trees  is not recognizable

$\rightsquigarrow$ tree automata with sibling equality constraints (Rec$_{\neq}$ [Bogaert&Tison'92])

- transitions: $(q_1, \ldots, q_k, \alpha, a, q)$
- α: Boolean combination of $t_i = t_j$, $t_i \neq t_j$ with $i, j \in \{1, \ldots, k\}$

  Example: "$t_1 = t_2$" means "left and right subtree are equal"
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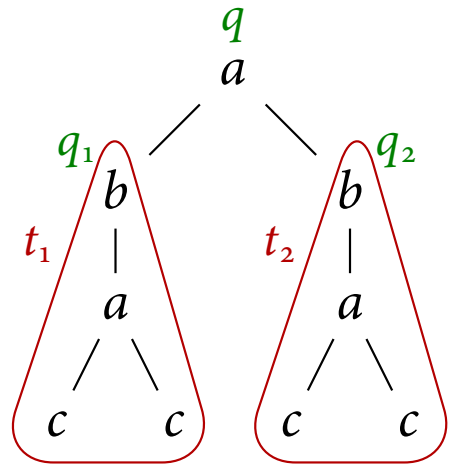

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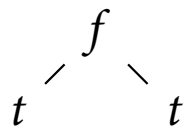
But language of trees  is not recognizable

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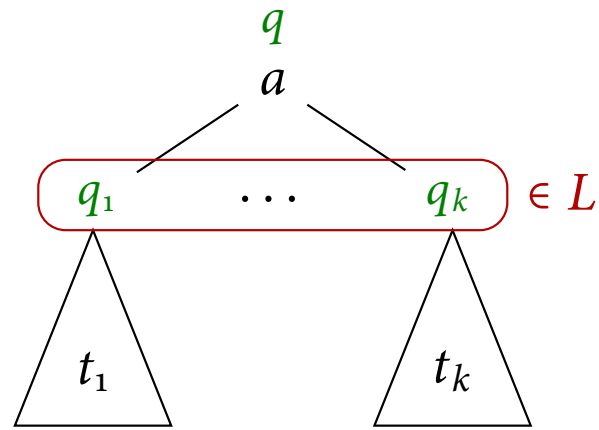
In this talk: extend the class Rec$_{\neq}$ to unranked trees

Motivations(2): Unranked Trees

Unranked trees:

- Formal model for XML documents
- Symbols have no fixed arities
- Number of successors of a node is unbounded
- Finite unranked tree automata: transitions (L, a, q) with L regular language over the state set



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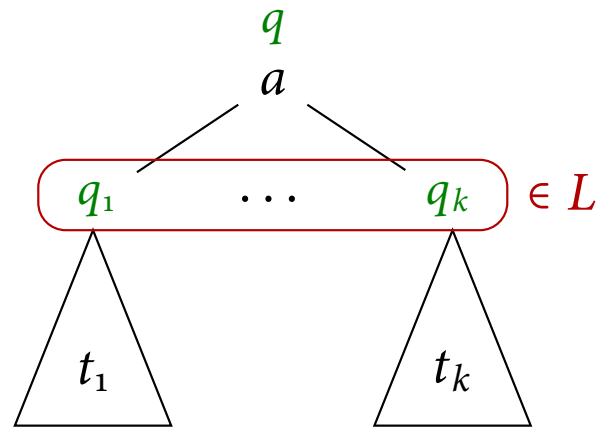
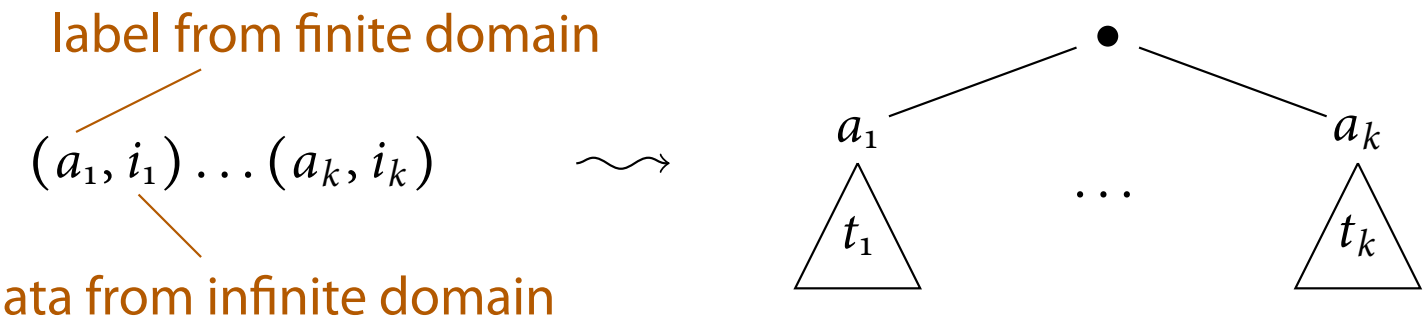


Subtree equality in unranked trees:

- Subtrees as data encoding (e.g. of natural numbers)

  For instance, data words (e.g. [Bojańczyk et al.'06]) can be coded as trees:



  Automata on data words: test equality between data
  $\rightsquigarrow$ data equalities $\approx$ subtree equalities

Outline

Unranked Tree Automata with Sibling (Dis)Equalities

Decidability of Emptiness: the Deterministic Case

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Constraints among Unboundedly Many Siblings

Transition (L, α, a, q) with $L \subseteq Q^*$ regular

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- Constraint α has to consider all possible number of successors.
- Use of regular $L \subseteq Q^*$ allows finite representation despite unbounded length.

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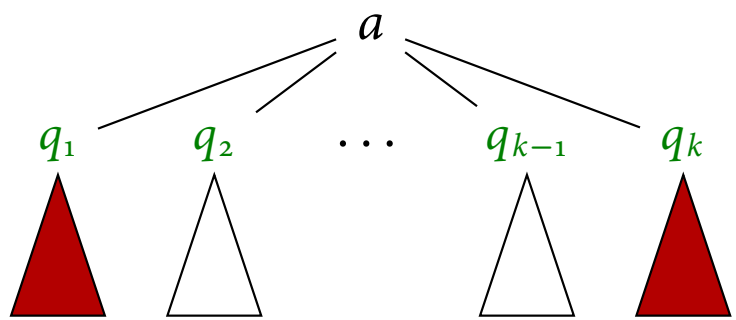

Say "first and last subtrees are equal, but different from the others"

- for fixed number of successors k: "$t_1 = t_k \wedge \bigwedge_{1<i<k} t_1 \neq t_i$"
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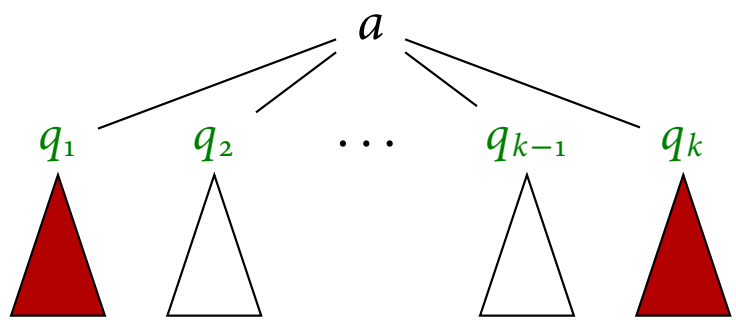

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Proposal: use logic formulas over Q-sequences

Using Logics over Q-Sequences

First-order (FO) / monadic second-order (MSO) logic over Q-sequences:

- interpreted in word structures $w = q_1 \dots q_k \in Q^+$
- FO-variables $x, y, z, \dots$ over positions in $\{1, \dots, k\}$
- MSO-variables $X, Y, Z, \dots$ over subsets of $\{1, \dots, k\}$
- $\varphi ::= x < y \mid x = y \mid X(x) \mid q(x) \mid \psi \vee \theta \mid \neg\psi \mid \exists x.\psi \mid \exists X.\psi$
- write $w \vDash \varphi$ for "w satisfies φ"

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Saying "first and last subtrees are equal, but different from the others" with logic formulas:

- intuitively: extend the vocabulary by subtree equality and say

$$\exists x \exists y \, (\, x = \min \wedge y = \max \wedge t_x = t_y \wedge \forall z \, (\, z \neq x \wedge z \neq y \rightarrow t_z \neq t_x \,) \,)$$

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- But this could lead to an automaton with undecidable emptiness since using trees we can encode data words, and satisfiability of FO logic over data words is undecidable [Bojańczyk et al.'06].

Constraints among Unboundedly Many Siblings – cont'd

Idea: separate addressing and subtree comparison
$\rightsquigarrow$ use formula only to address pairs of positions to be compared

Atomic constraint: $(\ \varphi(x, y)\ ,\ type\ )$

Constraint types: $\exists_{\mathsf{EQ}}\ ,\ \forall_{\mathsf{EQ}}\ ,\ \exists_{\mathsf{NEQ}}\ ,\ \forall_{\mathsf{NEQ}}$

$\varphi(x, y)$: MSO-formula with two free variables

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Semantics: $w = q_1 \dots q_k \in Q^*$ and tree sequence $t_1 \dots t_k$ satisfy $(\varphi, type)$ if:

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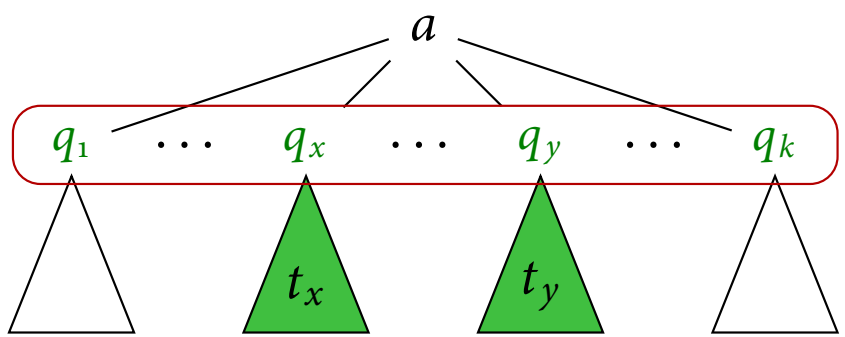

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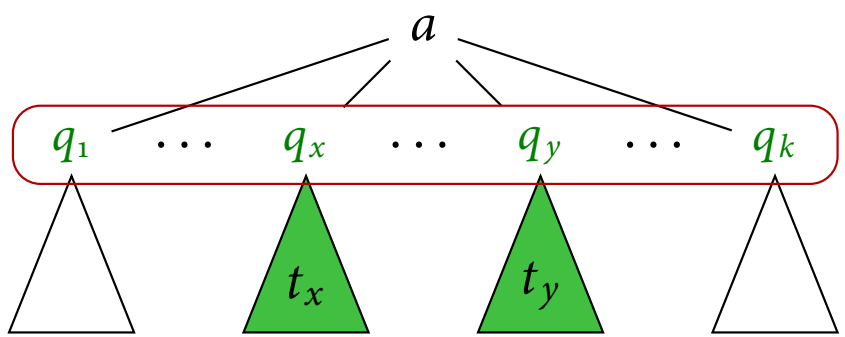

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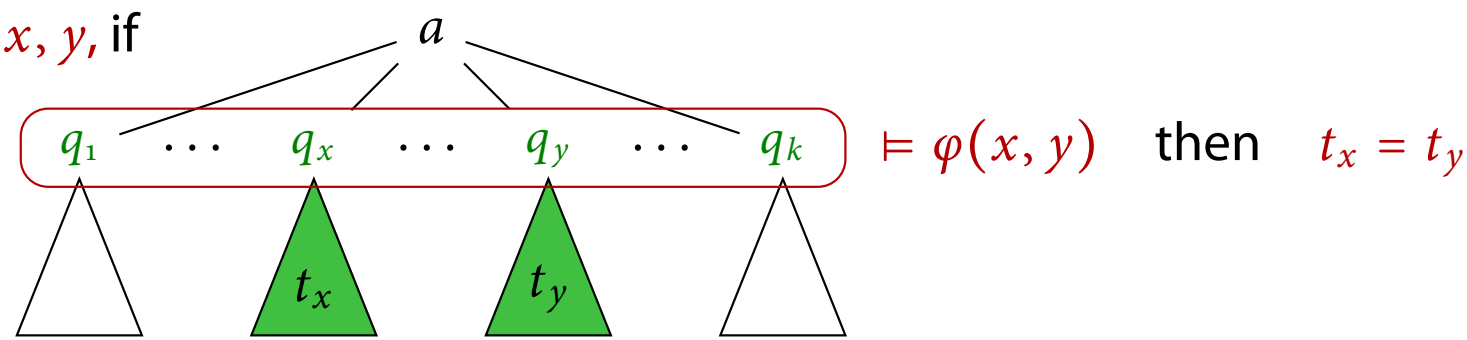

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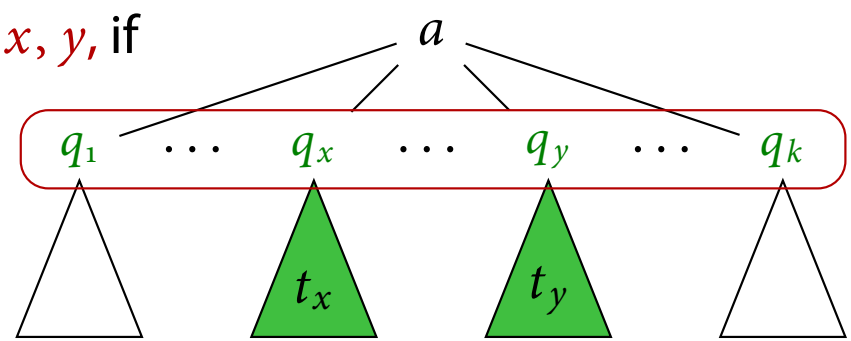

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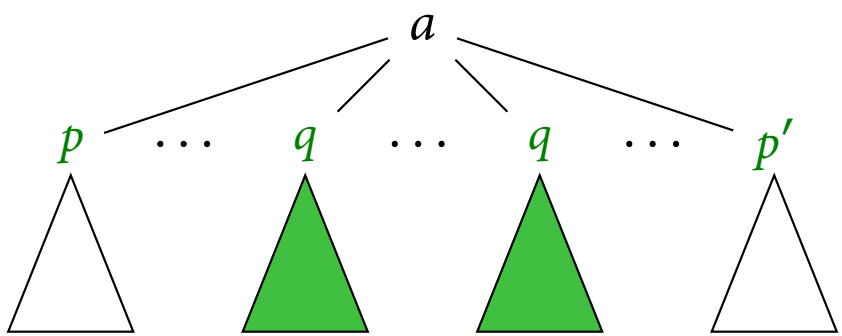
Constraints among Unboundedly Many Siblings – Examples

Constraints: Boolean combinations of atomic constraints $(\varphi, type)$

Example: "There are at least two positions labeled with q, and all positions labeled q have equal subtrees, and all other positions have distinct subtrees"



Express this by conjunction $(\varphi, \exists_{\mathsf{EQ}}) \wedge (\psi, \forall_{\mathsf{EQ}}) \wedge (\theta, \forall_{\mathsf{NEQ}})$ where:

$$\begin{aligned} \varphi(x,y) &:= \quad x < y \wedge q(x) \wedge q(y) \\ \psi(x,y) &:= \quad q(x) \wedge q(y) \\ \theta(x,y) &:= \quad q(x) \wedge \neg q(y) \end{aligned}$$

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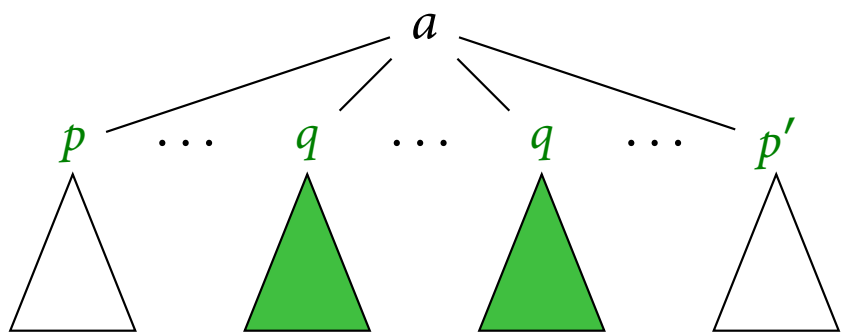

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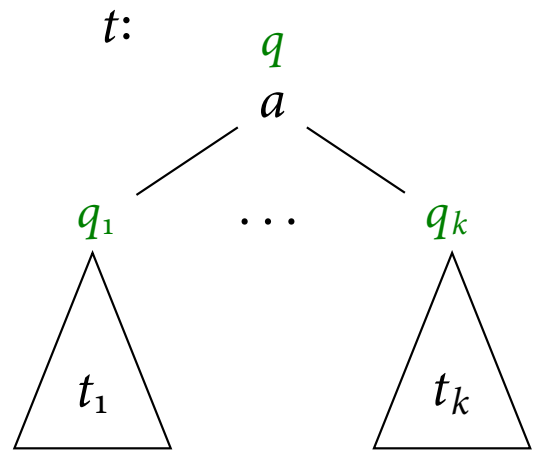
Remark:

- MSO-formulas only used as addressing mechanism
- no reuse of formulas in other constraints allowed

UTACS

Unranked Tree Automaton with Constraints between Siblings $\mathcal{A} = (Q, \Sigma, \Delta, F)$:

- finite, unranked alphabet Σ
- state set Q and set F of accepting states
- run: bottom-up assignment of states to nodes
- transition (L, α, a, q) with
  - $L \subseteq Q^*$ regular
  - α is Boolean combination of atomic sibling constraints $(\varphi, type)$



Some results from the ranked case directly transferable, e.g.:

- closure under union and intersection

Determinism vs. Nondeterminism

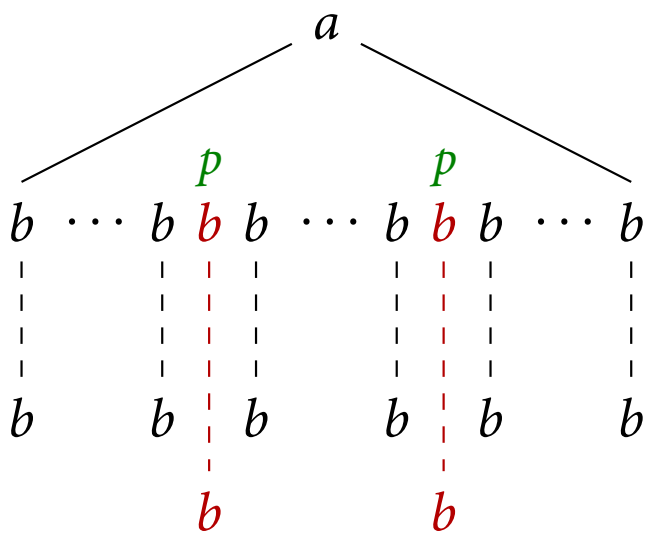
UTACS $\mathcal{A} = (Q, \Sigma, \Delta, F)$ is called deterministic if for each Σ-tree t there is at most one state q with $t \to_{\mathcal{A}} q$.

Proposition

Deterministic UTACS $\subsetneq$ Nondet. UTACS

Separating language contains trees of this form:

- root labeled with a
- root's children are strands of b's
- all but two b-strands are equal, and the latter two are equal



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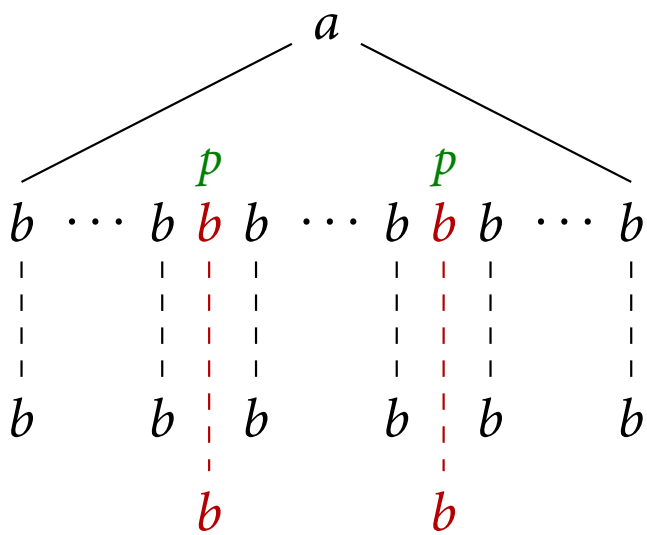

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Remark: Determinization possible if atomic constraints do not make use of Q (via subset construction)

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Decidability of Emptiness: the Deterministic Case

Deciding Emptiness

Generic emptiness algorithm for bottom-up tree automaton:

- Check whether some final state q is reachable, i.e., there is at least one tree t with $t \to_{\mathcal{A}} q$.
- Procedure:
  1. Keep track of reachable states and the trees evaluating to them.
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But more work is needed to handle (dis)equality constraints:

- If $t_2 \neq t_3$ is required:
  $\rightsquigarrow$ trees must be distinct whenever the states are
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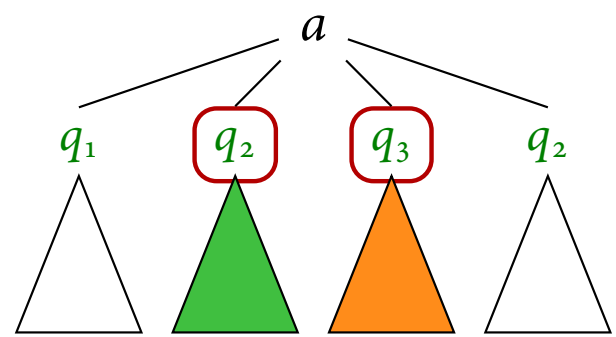

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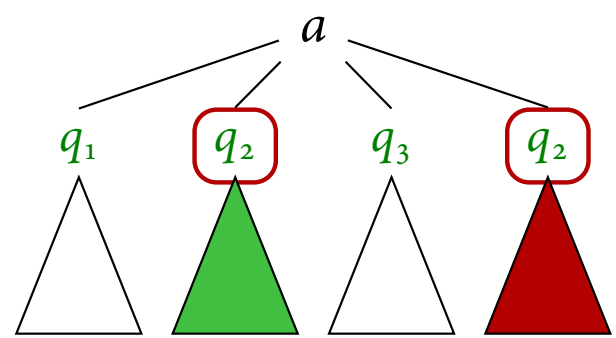
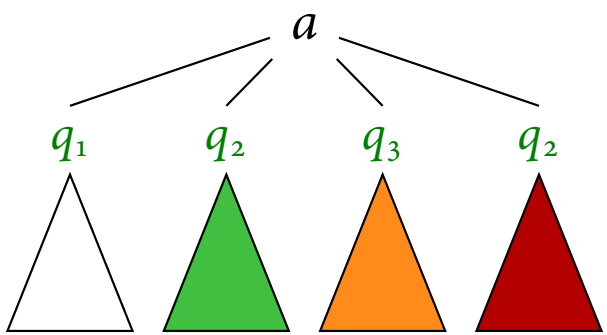


- If "$t_2 \neq t_4$" is required:
  $\rightsquigarrow$ transition can only be applied if there are at least two trees evaluating to q_2.
  $\rightsquigarrow$ for each state, a certain number of trees evaluating to it are needed

… But How Many Trees to Collect?

Ranked case: number of distinct trees needed $\leq$ maximal rank
$\rightsquigarrow$ bound on the number of trees to collect



Unranked case: …

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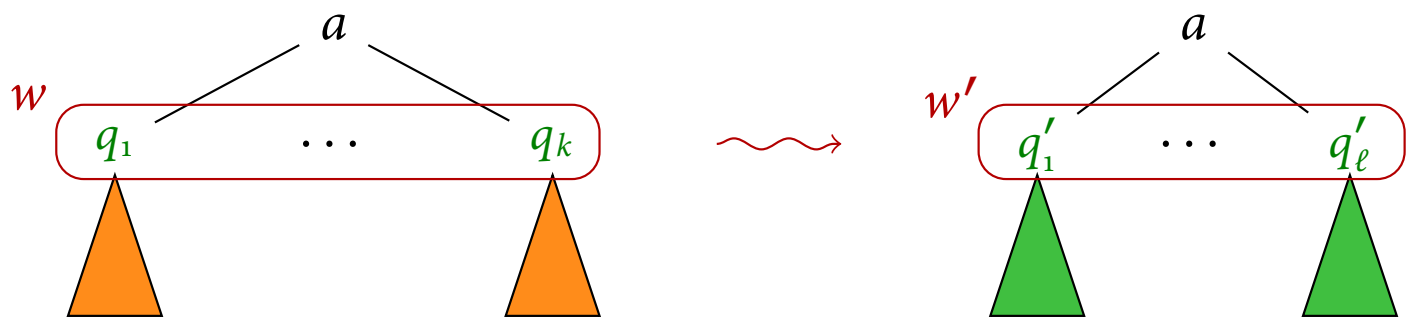
Lemma (Bound Lemma)

There exists a bound N such that: for each application of a transition $\tau = (L, \alpha, a, q)$ using $w = q_1 \dots q_k$, there is a replacement $w' = q'_1 \dots q'_\ell$ such that the application of τ using w' needs $\leq N$ distinct trees for each state.



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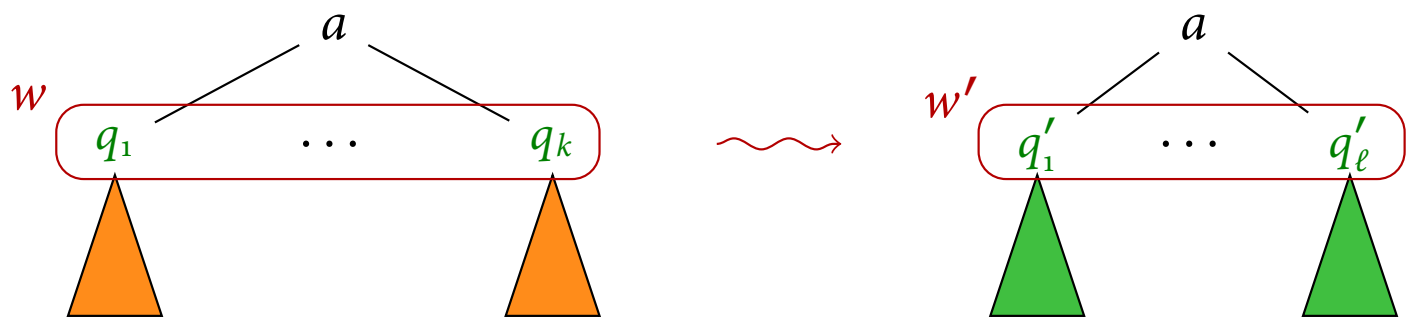
Unranked case: …

Lemma (Bound Lemma)

There exists a bound N such that: for each application of a transition $\tau = (L, \alpha, a, q)$ using $w = q_1 \dots q_k$, there is a replacement $w' = q'_1 \dots q'_\ell$ such that the application of τ using w' needs $\leq N$ distinct trees for each state.



Thus, emptiness algorithm needs to collect $\leq N$ distinct trees for each state.

Theorem: *The emptiness problem for deterministic UTACS is decidable.*

Conclusions

- Extension of tree automata with subtree (dis)equalities to unranked setting
- Use of MSO formulas as constraint addressing mechanism
- Decidability of emptiness for deterministic automata

$\rightsquigarrow$ First step to studying comparisons between data with tree representation

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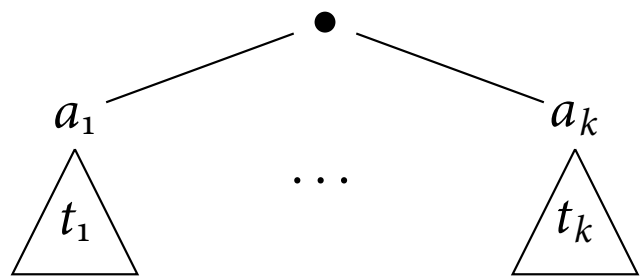
Next: compare the outputs of a preprocessing instead of the subtrees themselves
E.g., when considering representation of data words as trees, ignore the actual roots of the subtrees, i.e. compare t_i's instead of $a_i(t_i)$'s.

$$(a_1, i_1) \dots (a_k, i_k) \rightsquigarrow$$



$\rightsquigarrow$ incorporate transducer into our automaton model

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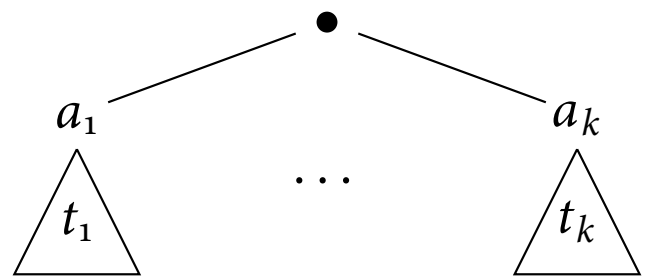

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Further prospects:

- nondeterministic UTACS
- relation to automata on data words