Nondeterministic Unranked Tree Automata with Sibling Equality Constraints

Karianto Wong

RWTH Aachen University

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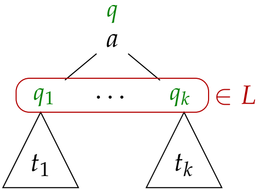
Joint work with Christof Löding

Unranked Tree Automata with Sibling Equalities

- Bottom-up tree automaton $\mathcal{A} = (Q, \Sigma, \Delta, F)$
- Finite state set Q with final states $F \subseteq Q$
- Symbols in Σ have no fixed arities
- Transitions:



$$(\, L \, , \, \alpha \, , \, a \, , \, q \,)$$

$L \subseteq Q^*$ regular

target state

current symbol

equality constraints between sibling subtrees, e.g.:

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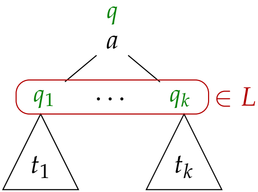

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equality constraints between sibling subtrees, e.g.:

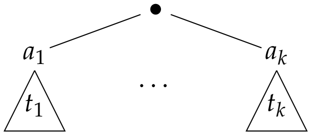
- “first=last”
- “all subtrees are equal”
- “first=last, but both are different from all others”

Motivations

- Natural extension of ranked tree automata with equality constraints between subtrees
  ⇝ [Mongy, Bogaert & Tison, Tommasi, … (in the 80's & 90's)]
- Unranked trees: formal model for semi-structured data
- Trees encode data (e.g. natural numbers)
  ⇝ data words can be coded as trees:

$$(a_1, i_1) \ldots (a_k, i_k) \quad \rightsquigarrow$$

label   data



Automata on data words: test equality between data (see, e.g., survey by [Segoufin'06])

⇝ data equalities ≈ subtree equalities

Outline

Automata with Constraints between Siblings Subtrees

Emptiness Problem

UTACS and Data Languages

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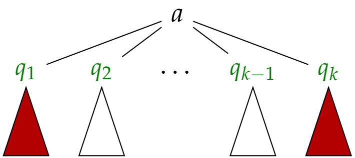
Constraints among Unboundedly Many Siblings

Symbols have no fixed arities $\rightsquigarrow$ unbounded number of sibling pairs to be compared

Example:

> "first and last subtrees are equal,
> but different from the others"



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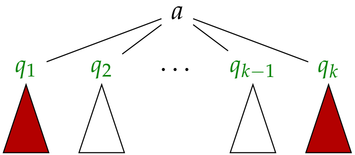

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First idea:

- use monadic second-order logic over state sequences:

$$\begin{aligned} \varphi \; ::= \quad & x < y \;\mid\; \mathrm{Succ}(x,y) \;\mid\; \mathrm{Lab}_q(x) \;\mid\; X(x) \\ & \mid\; \psi \vee \theta \;\mid\; \neg\psi \;\mid\; \exists x.\psi \;\mid\; \exists X.\psi \end{aligned}$$

- extend the vocabulary by subtree equality; e.g.:

$$\exists x \exists y \,(\, x = \min \wedge y = \max \wedge t_x = t_y \wedge \forall z \,(\, z \neq x \wedge z \neq y \;\rightarrow\; t_z \neq t_x \,)\,)$$

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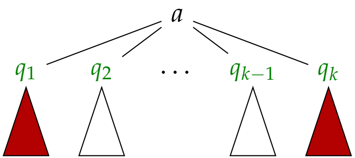

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$\rightsquigarrow$ Emptiness is undecidable since using trees we can encode data words, and satisfiability of FO logic over data words is undecidable [Bojańczyk et al.'06].

Constraints among Unboundedly Many Siblings – cont'd

Idea: separate addressing and subtree comparison

$\rightsquigarrow$ use MSO-formula only to address pairs of positions to be compared

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Four types of atomic sibling constraints:

- $\forall x \forall y \,.\, \varphi(x,y) \;\rightarrow\; t_x = t_y$

$\varphi(x,y)$: MSO-formula with free x, y

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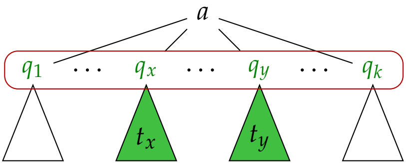

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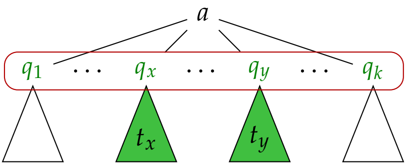

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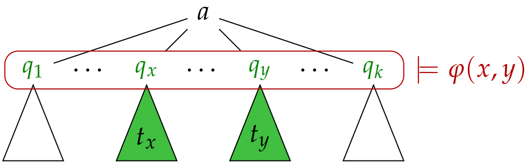

for some x, y

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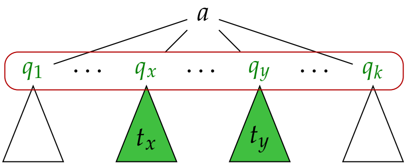

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Sibling constraints: Boolean combinations of atomic constraints

Example: "first and last subtree are equal, but different from the others"

$$\big[\, \forall x \forall y \,.\, x = \min \wedge y = \max \;\rightarrow\; t_x = t_y \,\big] \wedge$$

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UTACS

Unranked Tree Automaton with Constraints between Siblings:

- Bottom-up tree automaton $\mathcal{A} = (Q, \Sigma, \Delta, F)$
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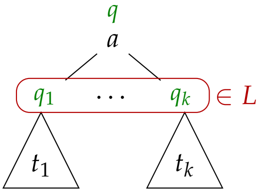

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equality constraints between sibling subtrees

Remarks:

- MSO-formulas only used as addressing mechanism
- No reuse of formulas in other constraints allowed

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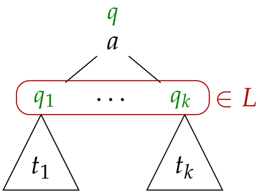

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Theorem [Löding&Wong'07]. *Nondeterministic UTACS are strictly more powerful than deterministic UTACS.*

Outline

Automata with Constraints between Siblings Subtrees

Emptiness Problem

UTACS and Data Languages

Emptiness Decidability

Theorem. *Emptiness for (nondeterministic) UTACS is decidable.*

In [Löding&Wong'07]: decidability for deterministic UTACS.

In this paper: decidability for nondeterministic UTACS.

- The methods used are basically the same as in deterministic case ...
- ... but a lot more technicalities are required.

Deciding Emptiness – the Deterministic Case

Generic emptiness algorithm for bottom-up tree automaton:

- Iteratively mark states reachable by some tree (and keep the tree).
- In each round: check whether some transition can be applied using trees that are currently available.

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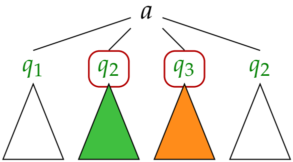
Checking applicability of transitions w.r.t. equality constraints:

- By determinism, reduce distinction between trees to distinction between states.

  $\rightsquigarrow$ If the states reached are different, then so are the trees

  Example: if "$t_2 \neq t_3$" is required, it suffices to know $q_2 \neq q_3$



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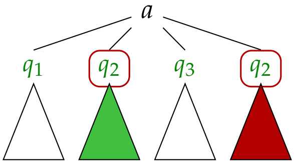
- For each state, collect a certain number of trees.

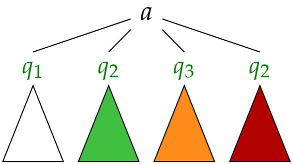
  Example: if "$t_2 \neq t_4$" is required, then the transition can only be applied if there are at least two trees evaluating to q_2.



... But How Many Trees to Collect?

Ranked setting: number of distinct trees needed $\leq$ maximal rank

$\rightsquigarrow$ bound on the number of trees to collect



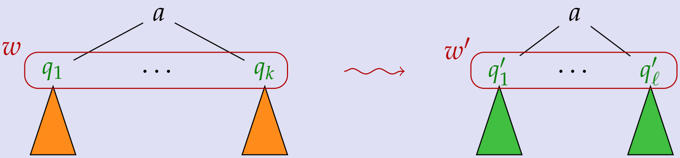
Unranked setting: ?

. . . But How Many Trees to Collect?

Ranked setting: number of distinct trees needed $\leq$ maximal rank
$\rightsquigarrow$ bound on the number of trees to collect

Unranked setting:

Lemma. *There exists a bound B such that: for each application of a transition $\tau = (L, \alpha, a, q)$ using $w = q_1 \ldots q_k$, there is a replacement $w' = q'_1 \ldots q'_\ell$ such that the application of τ using w' needs $\leq B$ distinct trees for each state.*

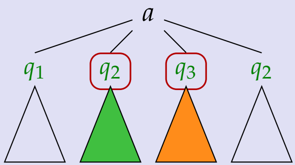


Remark: Our proof yields only non-elementary upper bound for B.

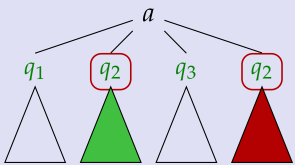
The Nondeterministic Case

Key observations in deterministic case: focus on the (unique) state reached by a tree

If the states reached are distinct, then so are the trees



For each state, a certain number of trees are needed



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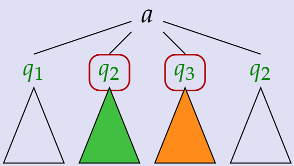

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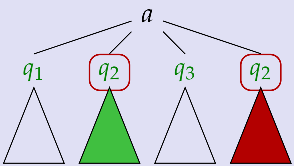

Nondeterministic case: pseudo-determinization directly in the algorithm

$\rightsquigarrow$ Proceed from states to sets of states!

Further ingredients: consider collections of transitions instead of single transitions

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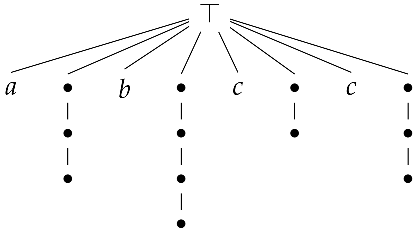
Encoding Data Words as Unranked Trees

label (finite alphabet)

$$(a,2)(b,3)(c,1)(c,2) \rightsquigarrow$$

data (infinite domain, e.g. $\mathbb{N}$)



- labels at odd positions
- data at even positions
- comparison between data values $\approx$ comparison between subtrees at even positions

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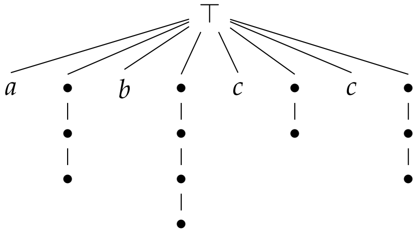

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$\rightsquigarrow$ Use UTACS to define languages of data words.

$\rightsquigarrow$ Emptiness is decidable for these languages of data words.

A Decidable Logic for Data Languages

Certain formulas of logic over data words (with data equality $\sim$) can be translated into UTACS, namely formulas corresponding to sibling constraints.

Example: “The data at first and last position are equal, but different from the data at the other positions”

$$\left[\ \forall x \forall y \,.\, x = \min \wedge y = \max \ \rightarrow\ x \sim y\ \right] \wedge$$

$$\left[\forall x \forall y \,.\, \big((x = \min \wedge x < y < \max) \vee (y = \max \wedge \min < x < y)\big) \rightarrow\ x \not\sim y\right]$$

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Another example: "between every two positions with the same data value, there exists a position labeled with a"

$$\left[\ \forall x \forall y \,.\, \neg\, \exists z \,.\, \big(\ x < z < y\ \wedge\ \mathrm{Lab}_a(z)\,\big) \rightarrow\ x \not\sim y\ \right]$$

$\rightsquigarrow$ It is still open whether this language is definable in $\mathrm{FO}^2(\sim, <, \mathrm{Succ})$ [Bojanczyk et al '06]

Conclusion

Summary:

- Use of MSO formulas as constraint addressing mechanism
- Emptiness is decidable
- Connection with languages of data words
- Universality is undecidable

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Open problems & further prospects:

- Complexity issues
- Comparing trees w.r.t. other relations, e.g., structural equality

Outline

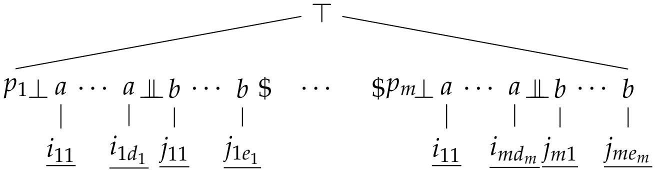
Appendix

Universality

Theorem. *Universality is undecidable for nondeterministic UTACS.*

Proof sketch:

- reduce the halting problem for 2-register machines
- encode computations $(p_1, d_1, e_1) \ldots (p_m, d_m, e_m)$ as a tree:



$\underline{i}$: unary tree representing natural number i

- construct nondeterministic UTACS accepting all trees that do not represent a halting computation

$\rightsquigarrow$ halting computation exists $\iff$ some tree is not accepted