From Solvability in λ-calculi to Meaningless Terms in TRSs

Vince Bárány

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Abstract

λ-calculus is one of the simplest formalisms imaginable, which is also suitable to represent computation as we know it. *Solvability* of λ-terms is a central notion related to the semantics of such representations, and the model theory of λ-calculus: it is proposed to capture exactly the *meaningful* terms; and it has a very interesting history.

Aside of their adequacy for representing the undefined, the unsolvable terms have some other valuable properties. Although a semantic notion solvability has a syntactical equivalent: the property of having a *head normal form*. Other results include the *genericity lemma*, the consistent identifiability of all unsolvable terms, and the fact that this is a maximal such set.

Motivated by these results joint efforts were made in the 1990's by Ariola, Kennaway, Klop, van Oostrom, Sleep and de Vries to characterize meaningless terms in the general framework of infinitary term rewriting systems (TRSs) and to establish axioms for a set of terms to possess some of the above mentioned desired properties of the unsolvables.

In his thesis J. Kuper took a different approach. He generalized solvability to a special simply typed $\lambda\delta$-calculus based on *strict contexts* derived from the rewriting rules. His technique and his notion of *usability* can as well be adapted and further generalized in an algebraic fashion to TRSs as I will show.

After a very brief introduction to λ-calculus I will display some of the basic properties of solvability and mention its role in the representation of partial recursive functions. I will also try to present the generalizations mentioned above pointing out some connections between the different approaches.