Tiling systems over infinite pictures and their acceptance conditions

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Abstract

Languages of infinite two-dimensional words (infinite ω-pictures) are studied in the automata theoretic setting of tiling systems. We show that a hierarchy of acceptance conditions as known from the theory of ω-languages can be established also over pictures. Since the usual pumping arguments fail, new proof techniques are necessary. Finally, we show that (unlike the case of ω-languages) none of the considered acceptance conditions leads to a class of infinitary picture languages which is closed under complementation.

1 Introduction

In the theory of automata over infinite words, many types of acceptance conditions have been studied, such as Büchi and Muller acceptance. In the framework of nondeterministic automata, three kinds of acceptance conditions have been singled out to which all other standard conditions can be reduced [11, 9, 2]: Referring to a nondeterministic automaton $\mathcal{A}$, an ω-word α is

1. A-accepted if some complete run of $\mathcal{A}$ on α exists,
2. E-accepted if some complete run of $\mathcal{A}$ on α exists, reaching a state in a given set F of final states,
3. Büchi-accepted if some complete run of $\mathcal{A}$ on α exists, reaching infinitely often a state in a given set F of final states.

It is well-known that these acceptance conditions lead to a strict hierarchy of three classes of ω-languages in the listed order.

The purpose of the present paper is to study these acceptance conditions over two-dimensional infinite words, i.e. labeled ω-grids or "infinite pictures". We use a model of "nondeterministic automaton" which was introduced under the name "tiling system" in [4] (see also the survey [3]). While the notion of run of a tiling system on a given infinite picture is natural, there are several versions of using the above acceptance conditions; for example one may refer to the occurrence of states on arbitrary picture positions, or one only considers the diagonal positions. As a preparatory step, we give a reduction to the latter case and thus use only the diagonal positions for visits of final states.

The first main result says that over infinite pictures we obtain the same hierarchy of languages as mentioned above for ω-languages. Whereas in the case of ω-words one can use simple state repetition arguments for the separation proofs, we need different arguments here, combining König's Lemma with certain boundedness conditions.

In the second part of the paper we show that the class of Büchi recognizable infinitary picture languages is not closed under complementation. We use a recursion theoretic result on infinitely branching infinite trees, namely that such trees with only finite branches constitute a set which is not in the Borel class Σ_1^1. From this we easily obtain a picture language that is not Σ_1^1 and thus not Büchi recognizable: One uses pictures which consist of a code of an infinitely branching finite-branch tree in the first row and otherwise contain dummy symbols. The hard part of the nonclosure result on complementation is to show the Büchi recognizability of pictures which code infinite-branch trees. For this, it is necessary to implement precise comparisons between an infinity of segments of the first row, using only finitely many states. It turns out that this is possible by using the "work space" $\omega \times \omega$.

This nonclosure proof should be compared with a corresponding result of Kaminski and Pinter [6] where a kind of Büchi acceptance is used over arbitrary acyclic graphs; in that case one has much more freedom to construct counter-examples and thus does not obtain the nonclosure result over pictures.

A related work on tiling problems over $\omega \times \omega$ appeared in [5]. There "dominoes" are placed on the ω-picture in a non-overlapping tiling, the information flow being realized by matching the colors of the domino boundaries. This difference is not essential; however in [5] only the unlabeled ω-picture is considered, and no picture languages are associated with the domino systems.

2 Tiling systems over infinite pictures

2.1 Preliminaries

Let Σ be a finite alphabet and $\hat{\Sigma} = \Sigma \uplus \{\#\}$. An ω-picture over Σ is a function $p : \omega \times \omega \to \hat{\Sigma}$ such that $p(i,0) = p(0,i) = \#$ for all $i \geq 0$ and $p(i,j) \in \Sigma$ for $i,j > 0$. (So we use $\# \notin \Sigma$ as a border marking of pictures.) $\Sigma^{\omega,\omega}$ is the set of all ω-pictures over Σ. An *ω-picture language* L is a subset of $\Sigma^{\omega,\omega}$.

We call the restriction $p \restriction_{\{0,\ldots,m\}\times\{0,\ldots n\}}$ to an initial segment of a picture p an *(m,n)-prefix picture* or just a *prefix* of p. If $n = m$ such a prefix is called a *square*. A *k-extension with* Σ of an (m,n)-prefix p is an $(m+k, n+k)$-prefix p' such that $p' \restriction_{\{0,\ldots,m\}\times\{0,\ldots n\}} = p$ and the vertices of p' outside p are labeled with Σ. Analogously we define an *ω-extension with* Σ of p. We write $p' = p \cdot^k \Sigma$ or $p' = p \cdot^\omega \Sigma$ if p' is a k-extension, respectively an ω-extension, of p by Σ.

We denote by $S_v = \{((i,j),(i{+}1,j)) \mid i,j \in \omega\}$ and $S_h = \{((i,j),(i,j{+}1)) \mid i,j \in \omega\}$ the vertical, respectively horizontal successor relation on ω^2.

A *path* in a picture p is a sequence $\pi = (v_0, v_1, v_2, \ldots)$ of vertices such that $(v_i, v_{i+1}) \in S_v \cup S_h$ for all $i \geq 0$. If $v_0 = (0,0)$ we call π an *initial path*, and if $\pi = (v_0, \ldots v_n)$ is finite we call π a path from v_0 to v_n. A vertex v_1 is *beyond* a vertex v_0 ($v_1 > v_0$) if there is a path from v_0 to v_1.

The *origin* of a picture p is the vertex $(0,0)$, the only "corner" of p. The *diagonal* of a picture p is the set of vertices $\mathrm{Di}(p) = \{(i,i) \mid i \in \omega\}$.

2.2 Tiling systems

A *tiling system* is a tuple $\mathcal{A} = (Q, \Sigma, \Delta, Acc)$ consisting of a finite set Q of *states*, a finite alphabet Σ, a finite set $\Delta \subseteq (\hat{\Sigma} \times Q)^4$ of *tiles*, and an acceptance component Acc (which may be a subset of Q or of 2^Q).

Tiles will be denoted by $\left(\begin{smallmatrix} a_1,q_1 & a_2,q_2 \\ a_3,q_3 & a_4,q_4 \end{smallmatrix}\right)$ with $a_i \in \hat{\Sigma}$ and $q_i \in Q$, and in general, over an alphabet Γ, by $\left(\begin{smallmatrix} \gamma_1 & \gamma_2 \\ \gamma_3 & \gamma_4 \end{smallmatrix}\right)$ with $\gamma_i \in \Gamma$. To indicate a combination of tiles we write $\left(\begin{smallmatrix} \gamma_1 & \gamma_2 \\ \gamma_3 & \gamma_4 \end{smallmatrix}\right) \circ \left(\begin{smallmatrix} \gamma'_1 & \gamma'_2 \\ \gamma'_3 & \gamma'_4 \end{smallmatrix}\right)$ for $\left(\begin{smallmatrix} (\gamma_1,\gamma'_1) & (\gamma_2,\gamma'_2) \\ (\gamma_3,\gamma'_3) & (\gamma_4,\gamma'_4) \end{smallmatrix}\right)$.

A *run* of a tiling system $\mathcal{A} = (Q, \Sigma, \Delta, Acc)$ on a picture p is a mapping $\rho : \omega^2 \to Q$ such that for all $i,j \in \omega$ with $p(i,j) = a_{i,j}$ and $\rho(i,j) = q_{i,j}$ we have $\left(\begin{smallmatrix} a_{i,j} & a_{i,j+1} \\ a_{i+1,j} & a_{i+1,j+1} \end{smallmatrix}\right) \circ \left(\begin{smallmatrix} q_{i,j} & q_{i,j+1} \\ q_{i+1,j} & q_{i+1,j+1} \end{smallmatrix}\right) \in \Delta$. One can view a run of $\mathcal{A}$ on p as an additional labeling of p with states from Q such that every 2×2 segment of the thus labeled picture is a tile in Δ.

2.3 Acceptance conditions

We consider different acceptance conditions for tiling systems, all of them similar to the well known ones from ω-automata over words. First consider the case where the acceptance component is a set $F \subseteq Q$ of states. A tiling system $\mathcal{A} = (Q, \Sigma, \Delta, F)$

- A-accepts p if there is a run ρ of $\mathcal{A}$ on p such that $\rho(v) \in F$ for all $v \in \omega^2$,
- E-accepts p if there is a run ρ of $\mathcal{A}$ on p such that $\rho(v) \in F$ for at least one $v \in \omega^2$,
- Büchi-accepts p if there is a run ρ of $\mathcal{A}$ on p such that $\rho(v) \in F$ for infinitely many $v \in \omega^2$,
- co-Büchi-accepts p if there is a run ρ of $\mathcal{A}$ on p such that $\rho(v) \in F$ for all but finitely many $v \in \omega^2$.

There are other natural acceptance conditions referring to an acceptance component *Acc* which is a set $\mathcal{F} \subseteq 2^Q$. We denote by $\mathrm{Oc}(\rho)$ the set of states occurring in ρ, and by $\mathrm{In}(\rho)$ the set of states occurring infinitely often in ρ. A tiling system $\mathcal{A}$ *Staiger-Wagner-accepts* (*Muller-accepts*) a picture p if $\mathrm{Oc}(\rho) \in \mathcal{F}$ ($\mathrm{In}(\rho) \in \mathcal{F}$) for some run ρ of $\mathcal{A}$ on p.

For any acceptance condition C, we say that a picture language L is *C-recognizable* if some tiling system $\mathcal{A}$ C-accepts precisely the pictures in L.

In the conditions above we consider a run on the whole picture and therefore call them *global acceptance conditions*. To emphasize this, we speak of picture languages which are globally A-recognizable, globally E-recognizable, etc. However, for our proofs it is convenient to look at more restricted conditions.

A tiling system $\mathcal{A} = (Q, \Sigma, \Delta, F)$ accepts a picture p with an *A-condition on the diagonal* if there is a run ρ of $\mathcal{A}$ on p such that $\rho(v) \in F$ for all $v \in \mathrm{Di}(p)$. Similarly we can define diagonal versions of all other acceptance conditions above by replacing $v \in \omega^2$ by $v \in \mathrm{Di}(p)$.

Theorem 1 *For every acceptance condition C from above, an ω-picture language L is globally recognizable with condition C if, and only if, L is recognizable with C on the diagonal.* □

The proof of this Theorem uses, independent of the acceptance condition, a powerset construction where at vertex (i, i) on the diagonal all states occuring at positions (i, j) and (j, i) for $j \leq i$ are collected. Due to space restrictions we have to skip the details here.

Using the same idea, we can reprove the standard simulation results for nondeterministic ω-automata over pictures, namely that nondeterministic Muller recognizability reduces to nondeterministic Büchi recognizability, and that nondeterministic co-Büchi and Staiger-Wagner recognizability reduces to E-recognizability.

Again, using a classical (product-) construction, we obtain:

Proposition 2 *For every acceptance condition C from above, the class of C-recognizable picture languages is closed under union and intersection.*

□

An alternative proof uses the results of the following subsection.

2.4 Monadic definability

Similarly as for ω-words and for finite pictures one verifies that every Büchi recognizable picture language can be defined by a by an existential monadic second order sentence (a Σ_1^1-formula). Here we view pictures p as relational structures over the signature $\{S_v, S_h, \leq_v, \leq_h, (P_a)_{a\in\Sigma}\}$ with universe ω^2, where S_v, S_h are are interpreted as the usual vertical and horizontal successor relations, and $\leq_v$ and $\leq_h$ as the corresponding linear orderings. We write $u < v$ if v is beyond u. $P_a v$ holds for a vertex $v \in \omega^2$ iff $p(v) = a$.

Proposition 3 *Let $\mathcal{A} = (Q, \Sigma, \Delta, Acc)$ be a tiling system. The picture language recognized by $\mathcal{A}$ with any of the acceptance conditions of Section 2.3 can be defined by an existential monadic second order sentence φ.*

Proof: The desired Σ_1^1-sentence φ describes (over any given picture p) that there is a successful run of $\mathcal{A}$ on p. For the details we refer only to Büchi acceptance, the other cases are easy variations.

Let $Q := \{1, \ldots, k\}$. We code states assigned to the p-vertices by disjoint subsets $Q_1, \ldots, Q_k$ of ω^2, Q_i containing the vertices where state i is assumed. The formula has to assure that the states are distributed in the picture according to the transition relation Δ. Furthermore we have to express that a state from F is assumed infinitely often. This can be done by expressing that there is an infinite sequence of vertices which strictly increases with respect to $\leq$ and which are labeled with the same state from F.

The following sentence φ describes the existence of a successful run of $\mathcal{A}$ on p:

$$
\begin{aligned}
\exists Q_1 \dots Q_k \exists R \quad \forall x \bigvee_{1\le i\le k} \big(Q_i x \wedge \bigwedge_{j\neq i} \neg Q_j x\big) \qquad (1)\\
\wedge\ \forall x_1 \dots x_4 \Big(S_h x_1 x_2 \wedge S_h x_3 x_4 \wedge S_v x_2 x_3 \wedge S_v x_3 x_4 \rightarrow \\
\bigvee_{\left(\begin{smallmatrix} a_1,q_1 & a_2,q_2 \\ a_3,q_3 & a_4,q_4\end{smallmatrix}\right)\in\Delta} \ \bigwedge_{1\le i\le 4} P_{a_i} x_i \wedge Q_{q_i} x_i\Big) \qquad (2)\\
\wedge \bigvee_{i\in F} \forall x\big(Rx \rightarrow Q_i x\big) \qquad (3)\\
\wedge\ \ \forall x \in R\ \exists y \in R\ \exists z\big(\neg z = x \wedge x \le_h z \wedge z \le_v y\big)
\end{aligned}
$$

□

3 Hierarchy results

Let us start with some natural examples of picture languages: Over the alphabet $\{a, b\}$, define

- $L_0 = \{b\}^{\omega,\omega}$ as the set of pictures carrying solely label b,
- L_1 as the set of all pictures containing at least one a,
- L_2 as the set of all pictures containing a infinitely often.

It is clear that L_0 is A-recognizable (by a tiling system which allows to cover only b-labeled vertices), that L_1 is E-recognizable (by a tiling system that visits a final state precisely at the a-labeled vertices), and that L_2 is Büchi recognizable (by the same tiling system applied with the Büchi condition). Let us show that L_1 is not A-recognizable and L_2 not Büchi recognizable:

Theorem 4 *(a) L_1 is not A-recognizable.*

(b) L_2 is not E-recognizable.

Proof: *(a):* Assume that $\mathcal{A} = (Q, \Sigma, \Delta, F)$ A-recognizes L_1. Consider the pictures p_i consisting of an (i, i)-prefix labeled b, and a everywhere else. Each p_i is A-accepted by $\mathcal{A}$. Let ρ_i be the partial accepting run on the (i, i)-prefix of p_i.

For the partial runs of $\mathcal{A}$ on the b-labeled (i,i)-prefixes we use the extension relation: ρ' on the b-labeled (j,j)-prefix extends ρ on the b-labeled (i,i)-prefix if $j > i$ and the restriction of ρ' to the (i,i)-square is ρ.

Via the extension relation, the partial runs ρ are arranged in a finitely branching tree, where the empty run represents the root and on level i all possible runs on the b-labeled (i,i)-square are collected. (Note that for each such run on level i there are only finitely many possible extensions on level $i+1$.)

By assumption the tree is infinite (use the runs ρ_i from above). So by König's Lemma there is an infinite path. It determines a run of $\mathcal{A}$ on the infinite picture which is labeled b everywhere. Contradiction.

(b): Assume $\mathcal{A} = (Q, \Sigma, \Delta, F)$ E-recognizes L_2. Two cases must be considered:

Case 1: There is a square prefix p allowing a run ρ of $\mathcal{A}$ with final state visited on p, and p can be extended to infinitely many square prefixes p' which beyond p are labeled solely with b such that ρ can be extended to a run ρ' on p'. Then, by König's Lemma, $\mathcal{A}$ E-accepts the picture consisting of p and b-labeled vertices everywhere else, contradicting our assumption.

Case 2: For any square prefix p allowing a run ρ of $\mathcal{A}$ with final state on p, there exist only finitely many extensions of p solely by b-labeled vertices to square prefixes p', such that ρ can be extended to a run ρ' on p'.

Let us consider one such square prefix p_0. For every possible run ρ with final state on p_0, let p_0^ρ be the largest possible square extension of p_0 only with b, such that ρ can be extended to a run ρ' on p_0^ρ.

Choose p'_0 as the largest of all p_0^ρ and let p_1 be the extension of p'_0 to a larger square in which extra columns and rows are added: first a row and column labeled b, then a row and column labeled a. (Note that a run assuming a final state on p_0 cannot be extended to any run on p_1.)

We repeat this this procedure with p_1. If there is no run with final state on p_1, we can directly set p_2 as the extension to the next larger square with an extra column and row of a-labeled vertices; otherwise, we apply the above mentioned construction to obtain p_2.

By iteration, we get a sequence $p_0, p_1, p_2, \ldots$ of square prefixes such that

- p_{i+1} is a square extension of p_i containing a's in the last column and row,

- there is no run of $\mathcal{A}$ on p_{i+1} with final state on p_i.

In the limit (the unique common extension of all p_i) we obtain an ω-picture p with infinitely many occurrences of a which does not admit a run with an occurrence of a final state. Contradiction. □

It is instructive to compare the proofs above with the corresponding arguments over ω-words. Regarding the set of ω-words over $\{a, b\}$ with at least one letter a, one refutes A-recognizability as follows: Assume an ω-automaton $\mathcal{A}$ with n states A-recognizes the language. Then it accepts $b^n a b^\omega$ and assumes a loop before the a (note that up to a already $n+1$ states are visited), allowing also to accept b^ω. A similar repetition argument applies to the set of ω-words with infinitely many a's. If it is E-recognized by $\mathcal{A}$ with n states, then it will accept $(b^n a)^\omega$ by a visit to a final state after a prefix $(b^n a)^i b^n$; and again via a loop in the last b-segment it will also accept $(b^n a)^i b^\omega$. Over pictures these simple constructions of runs from loops cannot be copied.

The situation for deterministic tiling systems is much easier. We mention these systems here only shortly. A tiling system is called *deterministic* if on any picture it allows at most one tile covering the origin, the state assigned to position $(i+1, j+1)$ is uniquely determined by the states at positions $(i, j), (i+1, j), (j+1, i)$, and the states at the border positions $(0, i+1)$ and $(j+1, 0)$ are determined by the state $(0, i)$, respectively $(j, 0)$. The classical Landweber hierachy (see [7, 10]) of ω-languages is defined using deterministic ω-automata with the acceptance conditions from Section 2.3. The hierachy proofs carry over without essential change to pictures, so we do not enter the details for deterministic tiling systems.

4 The complementation problem

It is easy (following the pattern of the well-known proofs over ω-words) to verify that the classes of A-recognizable and of E-recognizable picture languages are not closed under complement. Over ω-words, the Büchi recognizable languages are closed under complement. We show here that this result fails over pictures. For this purpose we use a well-known result of recursion theory, namely that the codes of finite-path trees form a set which is not Σ_1^1.

The *ω-trees* considered in this context are (possibly) infinitely branching. For technical purposes it is convenient to work with a coding of trees where the nodes are represented by nonempty sequences of positive integers, the sequence (1) representing the root, and a sequence $(1, i_2 \ldots, i_k)$

representing a node on level $k-1$. We do not require that the sons $(1, i_2, \ldots, i_k, j)$ of a node $(1, i_2, \ldots, i_k)$ have j-values which form an initial segment of the positive integers. So we identify an ω-tree with a nonempty prefix-closed set of sequences $(1, i_2, \ldots, i_k)$ with positive integers i_j. An ω-tree is called *finite-path* if all paths from the root are finite.

In a natural way, we use a unary coding (over the alphabet $\{1\}$) of numbers and code an ω-tree t by an ω-word over $\{1, \$\}$, taking \$ as a separation marker. A node $(i_1, i_2, \ldots, i_k)$ is encoded as the finite word $1^{i_1}\$1^{i_2}\$\ldots\$1^{i_k}\$\$$. The tree t itself is encoded by a concatenation of the encodings of all its nodes, with the restriction that the encoding of any given node must be preceded by the encoding of its father. In addition, we begin the whole encoding with an extra \$\$. Finite trees are encoded by ω-words by repeating the encoding of a node infinitely often.

Let T_1 be the set of ω-pictures over the alphabet $\{0, 1, \$\}$ which contain a code of an ω-tree in the first row and are labeled with 0 on the remaining positions. Let T_2 be the subset of T_1 of those pictures where the coded tree contains an infinite path.

Theorem 5 *The class of Büchi recognizable ω-picture languages is not closed under complement. In particular $T_2 \subseteq \{0, 1, \$\}^{\omega,\omega}$ is Büchi recognizable, but its complement is not.*

Proof: We use a standard result of recursion theory (see, e.g. [8, Sect. 16.3, Thm. XX]), saying that the set FT of finite-path trees is Π^1_1-complete. Thus it is not Σ^1_1-definable in second-order arithmetic. This implies in particular that the ω-picture language $T_1 \setminus T_2$ containing the corresponding tree codes is not definable by a monadic Σ^1_1-sentence as introduced in Section 2.3., and hence by Proposition 3 is not Büchi recognizable.

In the next Lemma we show that T_1 and T_2 are Büchi recognizable. Assuming that this class of picture languages is closed under complement, we get that $\{0, 1, \$\}^{\omega,\omega} \setminus T_2$ is Büchi recognizable. Hence by Proposition 2 the set $T_1 \setminus T_2$ of finite path trees, which is $(\{0, 1, \$\}^{\omega,\omega} \setminus T_2) \cap T_1$ would be Büchi recognizable, too. $\square$

Lemma 6 *(a) The language $T_1 \subseteq \{0, 1, \$\}^{\omega,\omega}$ of all pictures encoding an ω-tree is Büchi recognizable.*

(b) The language $T_2 \subseteq \{0, 1, \$\}^{\omega,\omega}$ of all pictures encoding an ω-tree with an infinite path is Büchi recognizable.

Proof: The idea for a tiling system which will recognize T_1 with Büchi-condition is to check increasing prefixes of a tree-encoding for correctness. An accepting run divides a picture into horizontal slices, one for every node. On every slice we test whether the father of the corresponding node, which will be guessed non-deterministically, has already been listed before. The tiles will not allow a node to be skipped from this procedure and the acceptance condition will require that these checks succeed infinitely often.

Every state of the tiling system will consist of three components. Since we will always have to know the encoding on the first line anywhere in the picture, we spend the first state component for the vertical propagation of this line.

The second and the third component of the states are used to check whether the father of the current node was guessed correctly. An typical accepting run checking 1\$1111\$\$ and 1\$1111\$111\$\$ is sketched in Figure 1, where only the third component of a corresponding state is shown. The correct beginning of an encoding including the root node has to be checked separately using another set of states.

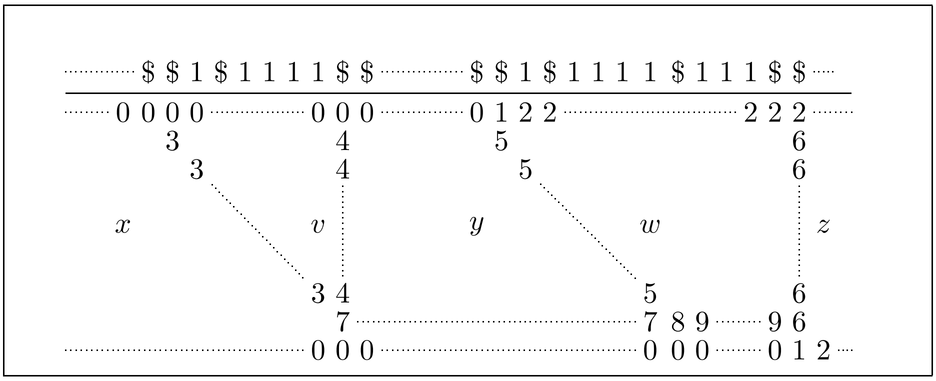

Figure 1: Accepting tiling of father and son

Let us outline the behaviour of $\mathcal{A}$ and the interpretation of each state. In the following we will use the term "labeling" for the third state component only.

The starting point of each comparison is the line consisting of 0,1 and 2 only. State 1 marks the end of the tree node checked last, everything to the left is labeled 0 (already checked), everything to the right is labeled 2 (to be checked).

In the next line we start the comparison of the node encoded just to the right of the vertex labeled 1. Therefore four signals are used, two diagonal ones from the beginning of the encoding of the father (state 3) and of the son (state 5), and two vertical ones (4 and 6) marking the their respective ends.

From every vertex (i, j) labeled 3 a signal to the right is initiated. This signal forwards the first state component of the state at (i, j) to the right using the second state component until it reaches a vertex $(i, j + k)$ labeled 5. The first state component of $(i, j+k)$ has to be the same as the second state component of the arriving signal for the run to be continued. This is the case only if vertices $(1, j)$ and $(1, j + k)$ are labeled with the same symbol.

We mark the vertex where signals 3 and 4 meet by 7, which is forwarded to the right to mark at its meeting point with signal 5 (labeled 8) the end of the enconding of the father contained in the encoding of the son. We use label 9 to check that henceforth only 1's appear in the encoding of the son. Finally, the vertex below the one where signals 9 and 6 meet is labeled 1, indicating that the node whose encoding ends here has been successfully verified. The labels x, v, y, w, z are used to distinguish the vertices contained in the fields bordered by signals 0-9.

To accept the language T_2 of ω-trees with an infinite path we modify the tiling system above slightly. We add another two state components. The first one is used to indicate (using state $*$) that the corresponding node is the last one which has already been verified and which is on the infinite path to be checked. The fifth component is used to forward this mark to the right when we verify the encoding of the son of this node on the infinte path. Once this son has been verified it becomes the only node labeled $*$ in the fourth component for slices below the current one.

The tiling system will still ensure that there is a run on a picture p if, and only if, p encodes an ω-tree. The acceptance condition now requires that the consistency check for nodes whose father is labeled with $*$ in the forth component succeeds infinitely often. □

5 Conclusion

In this paper we have isolated those aspects of acceptance of ω-pictures by tiling systems which differ from the theory of ω-languages. This concerns the proofs (but not the results) in the comparison of acceptance conditions. For the class of Büchi recognizable picture languages we showed the non-closure under complementation.

Among the many questions raised by this research we mention the following: Find (or disprove the existence of) decision procedures which test Büchi recognizable picture languages for E-, respectively A-recognizability. Compare the tiling system acceptance with an acceptance of pictures row by row using an automaton model over ordinal words of length ω^2 (see [1]). Finally, it would be nice to have elegant characterizations of the A- and E-recognizable picture languages which do not use the obvious restrictions in Part (3) of the formula in Proposition 3.

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